Theoretical Problem No. 3 (10 points)

”Squeezing” electrical charge carriers using magnetic fields

Plasma physics has to solve the problem of achieving devices capable of producing energy on a large scale through fusion. There are no issues related to the feasibility of the scientific method - the process can be observed as it happens in stars. But there are many technological feasibility problems primarily related to heating the plasma, and controlling it, at temperatures like the ones in the Sun. Physical conditions of the nuclear fusion cannot be achieved in containers. Maintaining plasma localized in limited volume can be accomplished using magnetic fields.

The first two tasks of the problem require analyzing several situations in which the movement of charged particles is limited by external magnetic fields. The third task asks you to study the confinement of electrical charge carriers through their own magnetic field. When solving the problem you may rely on the following numerical values: elementary electric charge $e = 1.6\times10^{-19}C$, mass of the electron $m = 9.1\times10^{-31}kg$, the magnetic permeability of vacuum $\mu_0 = 4\pi\times10^{-7}F\cdot m^{-1}$.

Task No. 1

A very long metallic cylinder (a perfect electric conductor), having the length L and the radius R, $(L >> R)$ rotates at constant angular speed ω around its axis of symmetry.

The cylinder is located in a homogeneous magnetic field whose induction $\vec{B}$ is parallel to the axis of symmetry of the cylinder. The mass of electron is m and his electric charge is $-e$. Let the dielectric permittivity of the material from which the cylinder is made be ε.

1.i. For a stationary situation determine the expression of the bulk density of electric charge ρ into the cylinder at a distance r from its axis of symmetry $(0 < r < R)$. Express the result as function of B, e, m, ω and ε. *(1.00p)*

1.ii. Determine the expression of angular velocity ω_0 so that the bulk charge density is zero at any point of the cylinder. Express the result as function of B, e and m. *(0.20p)*

1.iii. Evaluate the possibility of the practical realization of a zero bulk charge density in any point of the metallic cylinder, when an experiment of the type described in question is carried out in the Earth's magnetic field in a place where the magnetic field induction is $B = 1{,}82\times10^{-5}T$. Briefly argue the answer. *(0.30p)*

Task No. 2

In a vacuum chamber there is a long, straight wire of negligible thickness made from a material with high electrical conductivity. An electrical current with the intensity of $I = 10A$ passes through the wire. Electrons are sent on a direction perpendicular on the wire, towards the wire. Their motion starts at the distance r_0 from the wire, with an initial speed v_0 much smaller than speed of light. The electrons cannot approach the wire at a distance less than $r_0/2$. Consider two reference frames – one being the laboratory system S.L. and the other being a mobile system S.M. that runs parallel to the wire with a constant speed v_0 in the direction in which the current flows through the wire.
Neglect the magnetic field of Earth.

2.i. Deduce the expressions of induction of the magnetic field produced by the current flowing through the wire in each of the two reference frames $\vec{B}_{S.L.}(r), \vec{B}_{S.M.}(r)$. *(1.00p)*

2.ii. Determine the expression of the difference between Lorentz forces in the two reference systems, $\vec{F}_{S.L.}(r)-\vec{F}_{S.M.}(r)$, forces acting on the electron sent to the wire. Express the result in terms of e, μ_0, I, r and v_0. *(1.50p)*

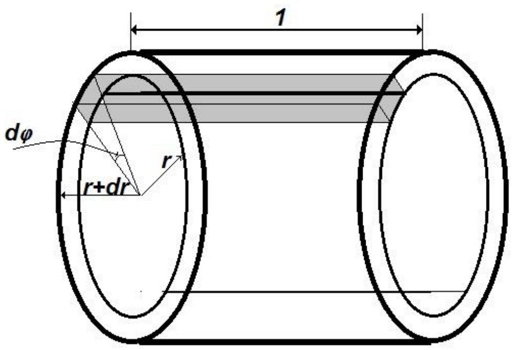
2.iii. State the values of the electron velocity's $\vec{v}(r_0/2)$ components in both reference frames. *(1.00p)*

2.iv. Determine the expression of the speed v_0 as function of e, m, I, μ_0. Calculate the numerical value of v_0. *(1.50p)*

2.v. Deduce the expression of the maximum distance from the wire D at which one can find the electron, as function of r_0, when the electron moves away from wire on a direction perpendicular on it. *(0.50p)*

Task No. 3

A cylindrical column of plasma having the radius R and the length L is generated in a vacuum chamber. The plasma appears as result of ionization of a gas, such that the concentrations of electrons $n_e(r)$ and ions $n_i(r)$ are equal at every point $n_i(r)=n_e(r)=n(r)$; the common value $n(r)$ depends only on the distance between the point and the axis of symmetry of the plasma cylinder. It is assumed that the plasma is in a stationary state, and therefore all its macroscopic characteristics are independent on time. It may be admitted that the temperature T of plasma is the same at every point of the column and that at these temperature the parameters describing plasma abide the perfect gas law. The electric charge of electron is $-e$ and the ions are monovalent, carrying an electric charge e. Consider as known the magnetic permeability of plasma μ and Boltzmann's constant k_B. Between the electrodes at the ends of the plasma column passes through the plasma an electrical current characterized by a constant density $j(r)=j$.



Consider an elementary portion of the hollow cylinder having the radiuses r and $r+dr$ as in joined figure. Elementary portion has a height equal to the unit. In the annulus (circular crown) representing the cross section of the plasma column the elementary portion covers the angle $d\varphi$.

3.i. Write the expressions of the forces acting on this elementary volume of plasma ($\vec{F}_p$ due to pressure $p(r)$ in the column of plasma, $\vec{F}_e$ due to the interaction with electrical charges carriers that there are in plasma , $\vec{F}_m$ due to the interaction with magnetic field of electrical current flowing through plasma). Write the equation describing the equilibrium of the considered elementary portion of plasma. *(1.20p)*

3.ii. Deduce the expression of pressure $p(r)$ in a point of the column of plasma. Express the answer as function of r , j , R and magnetic permeability μ. *(1.00p)*

3.iii. Determine the expression of the number of particles N carrying electrical charge in the column of plasma as function of L , I , T , μ și k_B. *(0.80p)*

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ANSWER SHEET

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Task No. 1

1.i. The expression of the bulk density of electric charge ρ into the cylinder at a distance r from its axis of symmetry		1.00p
1.ii. The expression of angular velocity ω_0 so that the bulk charge density is zero at any point of the cylinder		0.20p
1.iii. Evaluate the possibility of the practical realization of a zero bulk charge density in any point of the metallic cylinder. Briefly argue the answer		0.30p

Task No. 2

2.i. The expressions of induction of the magnetic field produced by the current flowing through the wire in each of the two reference frames $\vec{B}_{S.L.}(r)$ and $\vec{B}_{S.M.}(r)$		1.00p
2.ii. The expression of the difference between Lorentz forces in the two reference systems, $\vec{F}_{S.L.}(r)-\vec{F}_{S.M.}(r)$		1.50p

2.iii. State the values of the electron velocity's $\vec{v}(r_0/2)$ components in both reference frames		1.00p
2.iv. The expression of the speed v_0 as function of e, m, I, μ_0 and the numerical value of v_0		1.50p
2.v. The expression of the maximum distance from the wire D at which one can find the electron, as function of r_0, when the electron moves away from wire on a direction perpendicular on it		0.50p

Task No. 3

3.i. The expressions of the forces acting on this elementary volume of plasma		0.80p
The equation describing the equilibrium of the considered elementary portion of plasma		0.40p
3.ii. The expression of pressure $p(r)$ in a point of the column of plasma		1.00p

3.iii. The expression of the number of particles N carrying electrical charge in the column of plasma		0.80p