Experimental Problem no.1 (10 points)

Determination of length, mass and density using a chronometer

This experimental problem I propose that - using a chronometer and a piece of paper calibrated in arbitrary units a.u. - to determine the geometric dimensions of a can and express the results in meters. Also, the problem propose that - using experimental measurements you've done on the can - to determine its mass in kilograms, and its density in SI units. For this you will be provided with an experimental set, consisting of:

A. A cylindrical tin closed at one end (a can).

B. A sheet of paper marked with equidistant lines; distance between the lines is NOT known.

C. A chronometer.

D. A very strong thread, inextensible, perfectly deformable and of negligible mass.

E. A plastic template contour of Romanian borders allowing a proper suspension of can.

F. A tank vessel containing chemically pure water (water is assumed ideal fluid without viscosity).

G. A plastic syringe with unknown internal volume; on the syringe are marked equidistant divisions. The volume of liquid removed from the syringe when the piston travel a division is the fifth part of the volume of the syringe. Water will be added in the can in "portions" corresponding to the volume of a division - considered as "elementary volume".

H. Double-sided scotch tape for mounting the template on table.

I. Absorbent paper towels.

J. A toothpick (of negligible mass) serving to yield the initial rotation of the can and to observe its oscillations.

Study small rotating oscillations of empty can and of the can that have added one or more elementary volumes of water. Considers that the gravitational acceleration is $g = 9{,}8\, m \cdot s^{-2}$ and that the density of water is $\rho_0 = 1{,}0 \times 10^3\, kg \cdot m^{-3}$. Write all data known or measured and all the results you get in different workloads using two significant figures.

Fill in the appropriate boxes in the answer sheet with your results.

In solving the problem uses the following notations:

L	- For the length of each of the two suspension wires
$a = r \cdot L$	- For the radius of the cover of can
$b = p \cdot L$	- For the height of the can
$t = q \cdot L$	- For sheet thickness of which the can was made
$v = \dfrac{\pi \cdot a^2 \cdot b}{N} = \dfrac{\pi \cdot r^2 \cdot p}{N} \cdot L^3$	- For „elementary” volume corresponding to a division marked on the syringe. In expression N is five time the number of syringe of water necessary to completely fill the can.
$\rho = \eta \cdot \rho_0$	- For unknown density of can material.
m	- For unknown mass of empty can.
m_1	- For the unknown mass of elementary volume of water v.

Keep in mind that $t << a$.

Consider the coordinate system $Oxyz$ shown in the figure below. When the can is in balance, the center of the circular upper face of the box is the origin O of the system; the Ox system axis passing through the two mounting holes of the suspension wires. Axis Oy is perpendicular to the axis. The Oz axis of the coordinate system is the axis of symmetry of the box.

Task no. 1 - Deduction of the expressions useful for solving experimental problem

Assume that the can rotates so that the centers of its circular faces remain on Oz axis. In the new situation, the direction of each wires forms angle φ with the vertical original direction. During rotation, the wires are always stretched; a diameter of circular face of can rotates with the angle α, and the can rises with h. Consider that one of the suspension wires is trapped in C. In Figure 1 is represented the evolution of the radius OC of the top face of the can.

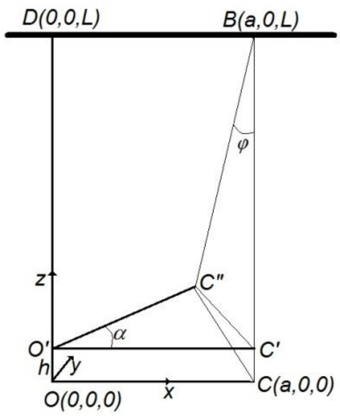

Figure 1

In the figure 1, OC is the radius of the top face of the can in the equilibrium position and $O'C"$ is the radius of the top face of the can when the box is rotated by the angle α around the axis of symmetry. $O'C'$ is a segment of length equal to the radius, parallel to OC and at a distance h of OC.

1.a. Write the expression for the coordinates of $C"$ - the place where a suspension wire is caught - in the situation where the box is rotated by the angle α around the vertical axis of symmetry. Express your answer in terms of, a, h and α.

1.b. Deduce the expression of angle φ as function on the angle α, length L and radius a of can cover .

1.c. Deduce the expression of angle φ and elevation h, when the angle α is small enough to admit that $\sin\alpha \cong \alpha;\ \cos\alpha \cong 1;\ \alpha^2 = 0$. Express the results as functions of, L, a and α.

1.d. Deduce the expression of the interior volume V, as well as the expressions for the mass m and moment of inertia J of the empty can. Express results as appropriate functions of L, r, p, q, η and ρ_0.

1.e. Determine the expression of the mass m_k and of moment of inertia J_k of the can filed with k "elementary volumes "of water. Express the results as appropriate functions of $L, r, p, q, \eta, \rho_0, N$ and k.

1.f. Deduce the expression of the momentum M_T of the forces acting on the can when it is rotated by a small angle α. Express your answer in terms of m_k, g, a, L and α.

1.g. Neglecting damping, determine the equation of motion describing the rotation of the can with a small angle α and show that it performs simple harmonic motion.

1.h. Period of oscillatory motion of the can has the expression $T_{oscil} = \frac{2\pi}{\Omega}$ where

$$\Omega^2 = \frac{g}{L^w} \cdot r^2 \cdot \frac{q \cdot \eta \cdot (r + 2p) + p \cdot r \cdot \frac{k}{N}}{r^2 \cdot q \cdot \left(\frac{r}{2} + 2p\right) \cdot \eta}$$

Find the unknown coefficient w in the above expression.

Task no. 2 – Measurements

2.a. Using paper marked in arbitrary unit (a.u.) determines the volume of metallic material of the can in arbitrary units. Arbitrary unit is the distance between two lines on paper sheet provided. On the chair there is a stack - compact packaging - of 10 metal sheets identical to that used for the can. You can use it if you need.

2.b. Using marked paper determines the volume of can in $(a.u.)^3$.

2.c. Determine the size of „elementary volume " in $(a.u.)^3$.

2.d. Measure the period of the rotating oscillation for the empty can and for the can having added one or more elementary volumes of water. Put the measured data in a table for each amount of water (measured time, number of oscillations and period)

Task no. 3 – Interpretation of experimental data

3.a. In the appropriate box in Answer sheet builds a linear plot of experimental data that allow you to determine the geometric dimensions of the box, its mass and density of its material. Express all these characteristics in SI units.

3.b. Use the graphical representation obtained in the task 3.a. to determine the geometric dimensions of the can *in meters*.

3.c. Using the graphical representation obtained in the task 3.a. determine the mass of the can *in kilograms*.

3.d. Calculate the density of the metallic material of the can and the elementary volume of water and express the results in SI.
3.e. Write the errors when measuring length in arbitrary units and for measuring time using chronometer. Decide whether the errors are larger when the amount of water increases. Justify your answer.

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Answer Sheet

Experimental Problem no.1 (10 points)

Determination of length, mass and density using a chronometer

Task no.1 - Deduction of the expressions useful for solving experimental problem

1.a. The coordinates of C'' - the place where a suspension wire is caught - in the situation where the box is rotated by the angle α around the vertical axis of symmetry		0,20p

1.b. Expression of angle φ as function on the angle α, length L and radius a of can cover

	0,60p

1.c. Expression of angle φ and elevation h, when the angle α is small enough

0,40p

1.d. Expressions of the interior volume V, as well as the expressions for the mass m and momentum of inertia J of the empty can

0,60p

1.e. Expressions of the mass m_k and of moment of inertia J_k of the can filed with k "elementary volumes "of water

	0,40p

1.f. Expression of the momentum M_T of the forces acting on the can when it is rotated by a small angle α

	0,80p

1.g. Equation of motion describing the rotation of the can with a small angle α showing that the can performs simple harmonic motion(when neglecting damping)

0,60p

1.h. The value of the unknown coefficient w

0,40p

Task nr.2 - Measurements

2.a. Calculation of the volume of metallic material of the can in $(a.u.)^3$

0,80p

2.b. Calculation of the volume of the can in $(a.u.)^3$

0,20p

2.c. Calculation of the elementary volume of water in $(a.u.)^3$

0,20p

2.d. Measurement of the periods of the rotating oscillation for the empty can and for the can having added one or more elementary volumes of water. Writing the measured data in a table for each amount of water (measured time, number of oscillations and period)

1,80p

2.d. (data)

Task no.3 – Interpretation of experimental data

3.a. Appropriate linear plot of experimental data allowing you to determine the geometric dimensions of the box, its mass and density of its material in SI units

1,80p

3.a. (further calculations)

3.b. Calculation of the dimensions of the can *in meters* using the graph obtained in task 3.a.

0,50p

3.c.. Calculation of the mass of the can *in kilograms* using the graph obtained in task 3.a.

0,20p

3.d. Calculations of density of the metallic material of the can and of the value of elementary volume of water v in SI units.

0,20p

3.e. The values of errors when measuring length in arbitrary units and for measuring time using chronometer. Decision if the error increases when the amount of water increases. Justification

0,30p